Analytic Johns Matrix and Its Applications in TLM Diakoptics

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Abstract – The storage of a general TLM Johns matrix is large because the dimension of Johns matrix is dependent on the iteration number. The time-domain convolution with Johns matrix is computationally intensive as the whole time history of the incident impulses is involved. In this paper, analytic Johns matrix is derived. A recursive convolution formulas is then developed, resulting in a considerable reduction of the computation count.

I. Introduction

The Transmission Line Matrix (TLM) method is a numerical modeling tool for solving electromagnetic field problem in space and time. An extensive list of references can be found in a chapter on TLM applications [1] .

Difficulties arise in the TLM method for the problems containing large areas with the changes only over a small part of geometry. Inefficiency results as the computation have to be re-iterated over the large unchanged areas every time a change is made.

The technique of diakoptics (or domain partitioning in time-domain) overcomes the disadvantages by breaking a solution domain down into smaller sub-domains. The subdomains are solved individually and then connected together to obtain the overall solution for a problem. The concept and idea were first presented by Johns [2, 3], further enhanced and generalized by Hoefer [4] with the introduction of the concept of Johns matrix, the discrete time-domain Green's function for TLM.

II. Analytic Johns Matrix

Consider a simple short-circuited rectangular waveguide stub as shown in Figure 1, modeled by a two-dimensional TLM network.

Conventionally, the Johns matrix at the open end is generated by first injecting a single unit impulse into branch 1, and computing the resulting impulse streams emerging from all M branches. This yields all elements $g_k(m,1)$ of the Johns matrix. Then branch 2 is excited, yielding all

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elements $g_k(m,2)$. In general, N such analyses or N runs of such numerical simulation are necessary to obtain all terms of the Johns matrix unless symmetry considerations allow us to reduce that number. The size of the Johns matrix would be $M \times N \times (K+1)$, which can be huge for a long iteration (large K).

With the Johns matrix, the response of the structure due to an arbitrary excitation in space and time can be computed by convolution:

$$V_k^r(m) = g_k(m,n) \otimes V_k^i(n) = \sum_{n=1}^{N} \sum_{k'=0}^{k} g_{k-k'}(m,n) V_{k'}^i(n) \qquad (1)$$

As can be seen, the convolution in time domain involves the whole time history of the incident impulses and the computation count increases as the k increases (time progresses). If a long iteration (large k) is needed, the computation will becomes extremely slow at the last few iterations as the CPU time for each iteration becomes too large.

Recent investigation has shown that the solutions of a recursive numerical method can be analytically expressed in terms of superposition of modal modes, each mode being weighted by exponential functions whose arguments are proportional to time or the time step k [3,4]. Since the Johns matrix is a particular TLM solution with special arrangements of the impulse injection as mentioned earlier, it can be written as:

$$g_k(m,n) = \sum_{p=1}^{P} b_{mnp}\, \lambda_p{}^k = \sum_{p=1}^{P} b_{mnp}\, |\lambda_p|^k\, e^{j\omega_p k \Delta t} \qquad (2)$$

where b_{mnp} is the constant independent of k and $\lambda_p = |\lambda_p|\, e^{j\omega_p \Delta t}$ is the eigenvalue of the modal matrix with ω_p being the modal frequency.

It should be noted that in the above equation, not all the terms need to be stored. If λ_p is a complex number, there must be an eigenvalue conjugate to λ_p. The same to b_{mnp}. Hence, conjugates of the λ_p and b_{mnp} need not be recorded. In addition, the structure (Figure 1) is lossy. As a result, the terms with $|\lambda_p|=1$, which represent non-decaying spurious modes, can be discarded.

III. Recursive Scheme for Convolution

With the analytic Johns matrix (2), the numerical convolution will become a simple recursive algorithm.

$$V_k^r(m) = g_k(m,n) \otimes V_k^i(n) = \sum_{n=1}^{N} \sum_{p=1}^{P} Q_{mnp}(k) \qquad (3)$$

where $Q_{mnp}(k) = \sum_{k'=1}^{k} b_{mnp}\, \lambda_p{}^{k-k'}\, V_{k'}^i(n)$

$= b_{mnp}\ V_k^i(m)\ +\ \lambda_p\ Q_{mnp}(k-1)$ with

$Q_{mnp}(0) = b_{mnp}\ V_0^i(m)$

In consequence, the numerical convolution does not require the storage of the complete time history of incident impulses. It is no longer dependent on the number of iteration. Thus, practical use of Johns matrix over a long iteration becomes feasible.

IV. Numerical Results

The rectangular waveguide cross-section $10\Delta l \times 16\Delta l$ is selected to validate and test the proposed method (see Figure 2). The structure was first divided into two separate sections $10\Delta l \times 9\Delta l$ and $10\Delta l \times 7\Delta l$ and the Johns matrices were generated using the conventional approach and the proposed method, respectively. Then, the Johns matrices were connected to obtain the solution.

Figure 3 shows the frequency responses of the waveguide obtained from the proposed method and from the conventional approach. They are basically the same.

Figure 4 presents the comparison between the computation time per iteration used by the conventional and proposed methods. For the initial period of simulation, the iteration number is small. The conventional approach takes less CPU time. However, as the iteration number increase, the computation amount per iteration in the conventional approach increases approximately linearly with the the iteration number (due to the convolution), while in the proposed method it is unchanged. As a result, when the iteration number exceeds 700, the proposed approach starts to have less computation time.

V. Conclusion

In this paper, an analytic Johns matrix has been obtained and consequently a recursive algorithm for convolution has been developed. It allows the practical applications of Johns matrix to realistic problems and the establishment of database for standard structures in the forms of Johns matrix.

References

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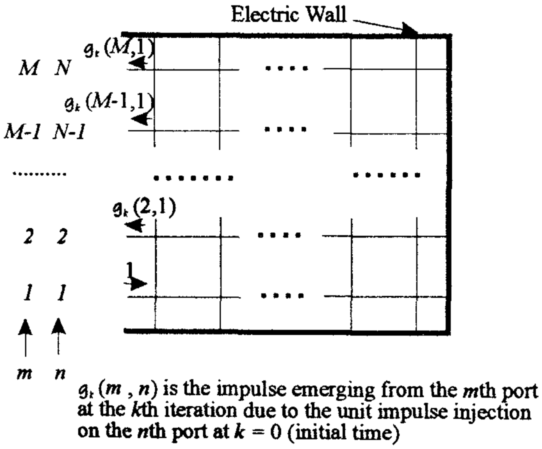

Fig.1 A short-circuited waveguide stub modelled by a two-dimensional TLM mesh

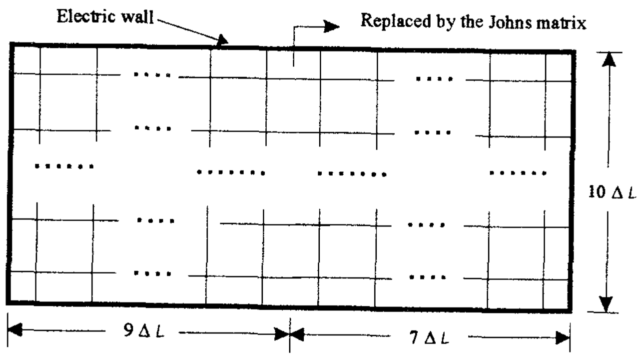

Fig.2 A rectangular waveguide cross-section 10 Δ L x 16 Δ L which is divided into two separate sections 10 Δ L x 9 Δ L and 10 Δ L x 9 Δ L

Fig.3 Frequency Response of the cut waveguide with convolution

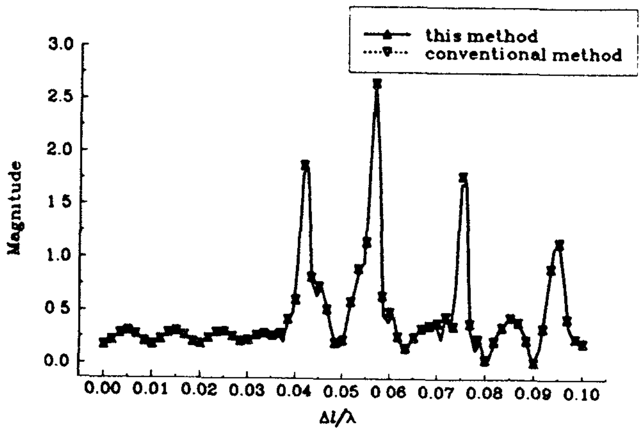

Fig.4 Average Computation time per iteration